Hence,

$$\phi(r,t) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho\left(r', t - \frac{|r-r'|}{c}\right)}{|r-r'|}\, d\tau \quad \text{--- (5)}$$

$$A(r,t) = \frac{\mu_0}{4\pi}\int_V \frac{j\left(r', t - \frac{r-r'}{c}\right)}{|r-r'|}\, d\tau \quad \text{--- (6)}$$

Are these the solutions of the inhomogeneous eq^n (2) and (3) ?

Let us substitute (5) into (2), we write (5) as

$$\phi(r,t) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(r', t - R'/c)}{R'}\, d\tau \quad \text{--- (7)}$$

$$R' = |r-r'|$$

It is difficult to evaluate the potential at $R' = 0$, we divide the volume of integration in eq^n (7) into two regions.

first → small sphere of radius r_0 around the point at which ϕ is to be determined

Second → rest of space.

The potential is given by

$$\phi = \phi_1 + \phi_2 \quad \text{--- (8)}$$

Hence $\nabla^2\phi = \nabla^2\phi_1 + \nabla^2\phi_2$

In the region of the small sphere $\frac{R'}{c} = \frac{|r-r'|}{c}$ is negligible, Hence

$$\nabla^2\phi_1 = \frac{1}{4\pi\epsilon_0}\nabla^2\int_V \frac{\rho(r', t)}{R'}\, d\tau = \frac{\rho}{4\pi\epsilon_0}\int_V \nabla^2\left(\frac{1}{R'}\right) d\tau$$

and $$\nabla^2\phi_1 = \frac{\rho}{4\pi\epsilon_0}\int_V \nabla^2\left(\frac{1}{R'}\right)d\tau \quad \text{--- (9)}$$

$$= -\rho/\epsilon_0 \qquad \left[\because \int_V \nabla^2\left(\frac{1}{R'}\right)d\tau = -4\pi\right]$$

Now, we find $\nabla^2\phi_2$.

We note that ∇^2 operates only on R'.
In spherical polar coordinates

$$\nabla^2\phi_2 = \frac{1}{R'}\frac{\partial^2}{\partial R'^2}(R'\phi_2) \quad \text{--- (10)}$$

$$= \frac{1}{4\pi\epsilon_0}\int_V \frac{1}{R'}\frac{\partial^2}{\partial R'^2}\rho\left(r', t-\frac{R'}{c}\right)d\tau \quad \text{--- (11)}$$

For any function $f(t - R'/c)$

$$\frac{\partial^2 f}{\partial R'^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} \quad \text{--- (12)}$$

therefore $$\nabla^2\phi_2 = \frac{1}{4\pi\epsilon_0}\int_V \frac{1}{R'c^2}\frac{\partial^2}{\partial t^2}\rho\left(r', t-\frac{R'}{c}\right)d\tau$$

$$= \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left[\frac{1}{4\pi\epsilon_0}\int_V \frac{\rho\left(r', t-\frac{R'}{c}\right)}{R'}d\tau'\right.$$

$$= \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} \quad \text{--- (13)}$$

$$\Rightarrow \nabla^2\phi = \nabla^2\phi_1 + \nabla^2\phi_2 = -\rho/\epsilon_0 + \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$

or $$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\rho/\epsilon_0$$

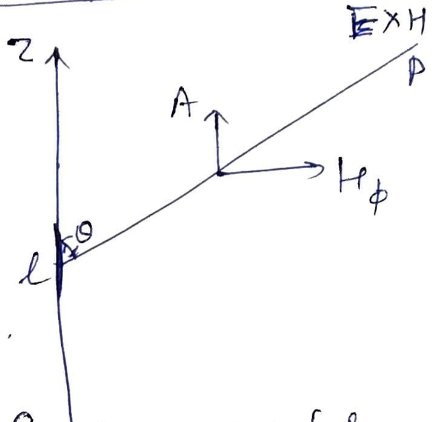
Thus eq^n(5) is a solution of eq^n(2).

These potentials (5) and (6) are called <u>retarded potentials</u>
Integrands $\rho(r')$ and $j(r')$ are to be evaluated at the retarded time $t - \frac{|r-r'|}{c}$, where $r' \to$ radius vector at the retarded time. This concept is familiar in astronomy

Radiation from an Oscillating Dipole

radiation field produced by an electric oscillating dipole?
This field has many important applications

Many practical radiating systems
→ may be considered to be
made up by putting together
a large number of such dipoles.



It contains many ~~[illegible]~~ features useful
in quantum theory of emission of radiation by atoms,
molecules and nuclei.

Consider two small spheres at the two ends
of a short wire. Suppose the charge is transferred
periodically from one sphere to the other, the time
variations being harmonic, i.e.

$$q = q_0 \exp(i\omega t) \quad \text{---} \quad (14)$$

q_0 → amplitude of oscillating charge.
ω → angular frequency of oscillation

Let us assume that the wavelength of the radiation
produced is large compared with the length of the
wire l, i.e.

$$\lambda = \frac{2\pi c}{\omega} \gg l \quad \text{or} \quad \frac{2\pi}{\omega} = T \gg \frac{l}{c}$$

This means that the time l/c taken for a signal to
propagate along the wire, from one end to the other, is
very much less than that over which the source current
changes appreciably. We may take the current I
to be same at all points along its length.